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PREDICTION OF AL_2O_3 LEACHING RECOVERY IN THE BAYER PROCESS USING STATISTICAL MULTILINEAR REGRESION ANALYSIS

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Abstract

This paper presents the results of defining the mathematical model which describes the dependence of leaching degree of Al_2O_3 in bauxite from the most influential input parameters in industrial conditions of conducting the leaching process in the Bayer technology of alumina production. Mathematical model is defined using the stepwise MLRA method, with $R^2 = 0.764$ and significant statistical reliability – $VIF<2$ and $p<0.05$, on the one-year statistical sample. Validation of the acquired model was performed using the data from the following year, collected from the process conducted under industrial conditions, rendering the same statistical reliability, with $R^2 = 0.759$.

Keywords: Prediction; MLRA; Stepwise; Leaching; Bayer process

1. Introduction

Bayer's process of alumina extraction from bauxite ore is the dominant process for obtaining alumina for more than 100 years. Today, more than 90% of alumina production in the world is achieved by this process, despite the fact that alternative processes have been developed in the meantime.[1, 2]

Bayer process involves the leaching of bauxite with concentrated sodium aluminate solution at temperatures between 100°C and 250°C, depending on the mineralogical form of the aluminum in bauxite. Trihydrate bauxite type – gibbsite can be dissolved in caustic solution in the temperature range

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100-180°C. Monohydrate bauxite forms (boehmite and diaspore) are dissolved in the temperature ranges: 130-180°C and 200-250°C, respectively [3]. This process includes reactions with soluble silica compounds and titanium dioxide under certain conditions. [4]

Process parameters influencing the leaching rate and the degree of Al_2O_3 recovery are: mineralogical and chemical composition of the bauxite, granulation size distribution, caustic modules of the starting solution and its Na_2O (caustic) content, temperature of the leaching process, stirring speed and duration of the process[5]. The process of bauxite leaching, under industrial conditions of the Bayer technology for alumina production, is highly complex. The ability to predict the recovery of Al_2O_3 during leaching, as the result of modeling the input parameters of the process, presents a large advantage for management of the process[4].

This paper presents the modeling of the leaching process of bauxite based on the results obtained under the industrial conditions, in order to predict the leaching degree of Al_2O_3 (process output) depending on the parameters of the process (process input). Obtained model presents a great advantage based on its ability to predict the accurate output of the investigated process.

2. Experimental data

The database used in this work was created on the basis of data collected from the industrial production in the alumina factory Birač in Zvornik (Bosnia and Herzegovina), whose capacity is 600,000 t of alumina per year. A complete set of data for all of the leaching process indicators had been measured daily during 2008. and 2009. and used to create a database consisting of 659 series of data taken under consistent operating conditions of the factory. Obtained database contains the chemical composition of the bauxite: Al_2O_3, SiO_2, Fe_2O_3, TiO_2, CaO, the loss on calcination, H_2O; composition of the aluminate solution at the beginning of the leaching process: Al_2O_3, Na_2O, caustic module at the beginning and the end of leaching, the chemical composition of autoclave residue: Al_2O_3, Na_2O_t, SiO_2, TiO_2, CaO, and the solution composition at the end of the leaching process: Na_2O_k and Al_2O_3. The “Al_2O_3 leaching recovery” – Y term refers to the alumina recovery in the autoclave leaching - digestion process and was calculated using the following equation:

$$Y(\text{ }Al_2O_3 \text{ leaching recovery}) = (1 - Al_2O_{3(am)} \cdot Fe_2O_{3(b)} / Al_2O_{3(b)} \cdot Fe_2O_{3(am)}) \cdot 100(\%) \quad ...(1)$$

Where:

$Al_2O_{3\ (b)}$, $Fe_2O_{3(b)}$ - the contents of components in bauxite (%)

$Al_2O_{3(am)}$, $Fe_2O_{3\ (am)}$ - component contents in the autoclave residue (%).

Using the formula (1) to calculate the leaching degree of Al_2O_3 by the use of “inert” Fe_2O_3 provides satisfactory results in industrial practice, with an accuracy of over 99%, from this reason it is used in this study.

For the modeling of the dependence of Al_2O_3 leaching degree from bauxite - Y (process output) using established database, following technological parameters of the leaching process (process inputs) were used:

X_1 - concentration of Na_2O_k in the leaching solution (g/l)

X_2 - caustic ratio of the solution at the beginning of the leaching process

X_3 - the moisture content of the bauxite (%)

X_4 - the Al_2O_3 content in the bauxite (%)

X_5 - the SiO_2 content in the bauxite (%)

X_6 - the Fe_2O_3 content in the bauxite (%)

X_7 - the TiO_2 content in the bauxite (%)

X_8 - the CaO content in the bauxite (%)

X_9 - the loss on calcination (%) and

X_{10} - caustic ratio of the solution at the end of the leaching process.

During the studied period, the database was generated from data measured during the industrial production in the factory Birač in Zvornik, during the stabile operation of the factory. At that time, as well as in the future period, processed bauxite mainly originated from the Vlasenica ore deposit (Bosnia and Herzegovina) which is of the boehmite type. Leaching temperature throughout the whole period was constant and equal to 245°C, with the pressure in autoclaves "reactors" kept to 35 bar. Granulation of the bauxite in all cases after hydrocyclonic classification was 100% - 74 μm, and the S : L ratio was 1 : 5. Rotational speed of the mechanical stirrers in the autoclaves was 31 rpm. Table 1 presents the experimental results in the form of descriptive statistics that were used for statistical modeling of process output-input dependence, i.e. $Y = f(X_1 - X_{10})$. A set of 300 data relates to the operation of the factory in 2008.

The values of standard deviations in Table 1 show that there is a normal Gaussian distribution with certain parameters. On this basis it can be concluded that this dataset is suitable for statistical analysis with multiple linear regression analysis (MLRA) technique. [5, 6] If the satisfactory degree of fitting is not achieved then the methods of nonlinear regression analysis (NLRA) are to be used, including methods of artificial neural networks (ANNs). [7, 8, 9]

Table 1. Descriptive statistics for the input (X_1 - X_{10}) and the output (Y) values of the bauxite leaching process in industrial process conditions for a set of 300 samples for 2008.

	N	Range	Minimum	Maximum	Mean		Std. Deviation	Variance
					Statistic	Std. Error		
X1	300	60.107	159.75	219.857	209.878	0.448	7.764	60.273
X2	300	1.701	2.075	3.776	3.089	0.012	0.215	0.046
X3	300	6.29	8.11	14.4	11.291	0.081	1.397	1.95
X4	300	3.54	50.13	53.67	51.734	0.045	0.775	0.6
X5	300	2.88	4.88	7.76	6.091	0.028	0.482	0.233
X6	300	3.64	23	26.64	24.846	0.039	0.669	0.447
X7	300	0.44	2.41	2.85	2.583	0.004	0.067	0.005
X8	300	2.68	0.43	3.11	1.331	0.021	0.358	0.128
X9	300	3.17	11.4	14.57	12.795	0.03	0.527	0.278
X10	300	0.284	1.355	1.639	1.404	0.001	0.025	0.001
Y	300	7.658	81.713	89.371	85.273	0.078	1.343	1.803
Valid N (listwise)	300	60.107	159.75	219.857	209.878	0.448	7.764	60.273

3. Statistical analysis

Forced-entry method was used to rank the influences of all predictors X_1 - X_{10} on the dependent variable Y, which in principle gives the possibility to define MLR dependence Y = f(X_1 - X_{10}), Table 2. The results indicate that the values of the variance inflation factor (VIF) are greater than two, indicating the existence of a larger share of collinearity among certain predictors. Also, the values of statistical significance ($p>0.05$) indicate that the Forced – entry method, in this case, does not provide statistically valid results.

Since the Forced-entry method did not give a satisfactory result based on the fact that it takes into consideration the impact of all predictors, it is useful to eliminate the impact of those predictors whose effect on output - Y is negligible. This requires the use of a stepwise regression analysis method, which solves the problem of collinearity by basing the order of the predictors entry on a mathematical criterion of ranking the individual predictors significance.[9] Acquired results of the stepwise method are shown in Table 3. Now the values of VIF are less than two, and the statistical significance is $p<0.05$, indicating a satisfactory statistical reliability of the results.

Redundant predictors were eliminated through the four step iteration, and the dependence of output – Y on the most important predictors: X_3, X_5, X_6 and X_{10} is defined with the following equation:

$$Y = 91.747 + 0.218X_3 - 1.496X_5 - 0.400X_6 + 7.206X_{10} \quad ....(2)$$

Fig.1 shows the comparison between the Y- measured and the Y-calculated values, with the coefficient of determination R^2 = 0.560.

Since the value of R^2 = 0.560 is determined by the distribution of errors above 4σ, Fig.2.a, in the further statistical analysis of this empirical data set, reduction of the statistical sample was carried out by

Table 2. MLRA values for a set of 300 samples, using the Forced-entry method

Model		Unstandardized Coefficients		Stand. Coeff.	t	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	46.440	17.063		2.722	0.007					
	X_1	-0.002	0.007	-0.012	-0.304	0.761	0.040	-0.018	-0.011	0.833	1.200
	X_2	0.421	0.310	0.068	1.357	0.176	0.441	0.080	0.051	0.561	1.782
	X_3	0.165	0.051	0.172	3.220	0.001	0.512	0.186	0.120	0.487	2.055
	X_4	0.446	0.182	0.257	2.451	0.015	0.527	0.143	0.091	0.126	7.934
	X_5	-0.980	0.214	-0.352	-4.575	0.000	-0.637	-0.260	-0.171	0.235	4.253
	X_6	0.070	0.194	0.035	0.359	0.720	-0.328	0.021	0.013	0.150	6.689
	X_7	0.863	0.826	0.043	1.045	0.297	0.304	0.061	0.039	0.816	1.226
	X_8	0.179	0.226	0.048	0.790	0.430	0.010	0.046	0.029	0.381	2.624
	X_9	0.627	0.215	0.246	2.913	0.004	0.341	0.169	0.109	0.195	5.133
	X_{10}	4.832	2.385	0.089	2.260	0.044	0.396	0.118	0.076	0.717	1.395

a. Dependent Variable: Y (%)

Table 3. MLRA values for a set of 300 samples, using the Stepwise method

Model		Unstandardized Coefficients		Stand. Coeff.	t	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	96.076	0.760		126.456	0.000					
	X_5	-1.774	0.124	-0.637	-14.264	0.000	-0.637	-0.637	-0.637	1.000	1.000
2	(Constant)	90.442	0.956		94.618	0.000					
	X_5	-1.481	0.117	-0.532	-12.644	0.000	-0.637	-0.592	-0.508	0.912	1.096
	X_3	0.341	0.040	0.355	8.432	0.000	0.512	0.439	0.339	0.912	1.096
3	(Constant)	102.689	2.489		41.262	0.000					
	X_5	-1.540	0.113	-0.553	-13.662	0.000	-0.637	-0.622	-0.526	0.903	1.107
	X_3	0.258	0.042	0.269	6.186	0.000	0.512	0.338	0.238	0.784	1.275
	X_6	-0.441	0.083	-0.220	-5.292	0.000	-0.328	-0.294	-0.204	0.860	1.163
4	(Constant)	91.747	4.258		21.549	0.000					
	X_5	-1.496	0.112	-0.537	-13.365	0.000	-0.637	-0.614	-0.507	0.889	1.124
	X_3	0.218	0.043	0.226	5.045	0.000	0.512	0.282	0.191	0.714	1.401
	X_6	-0.400	0.083	-0.199	-4.813	0.000	-0.328	-0.270	-0.182	0.839	1.193
	X_{10}	7.206	2.292	0.133	3.144	0.002	0.396	0.180	0.119	0.803	1.246

a. Dependent Variable: Y (%)

removing the extremes exceeding the 4σ boundaries. After ten iterations, 55 elements were discarded and the initial set was reduced to 245 elements with the distribution error within 4σ boundaries, and undisturbed statistical reliability Fig.2.b.

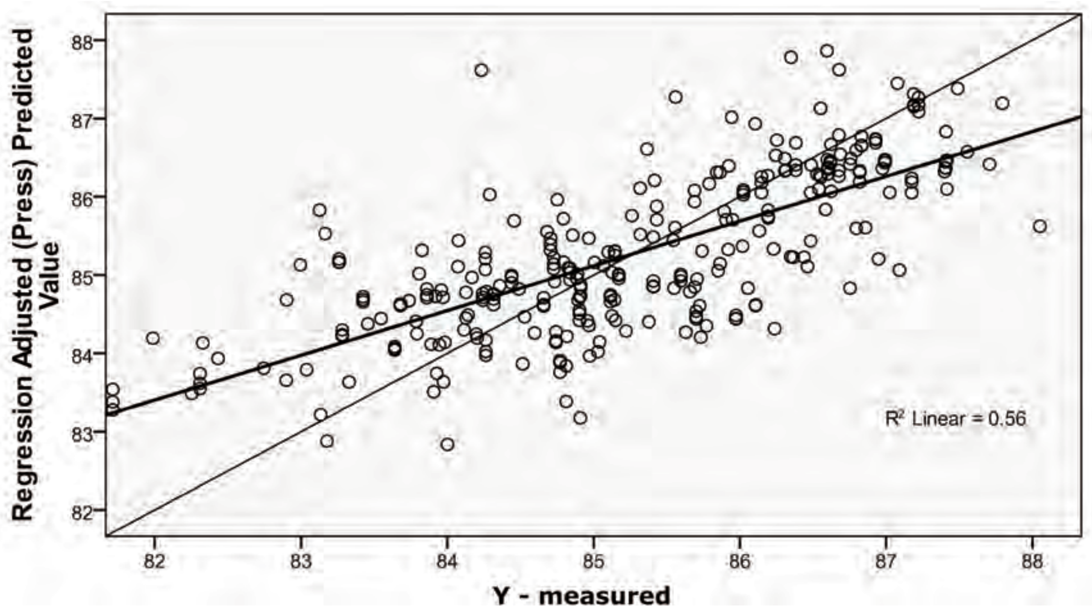
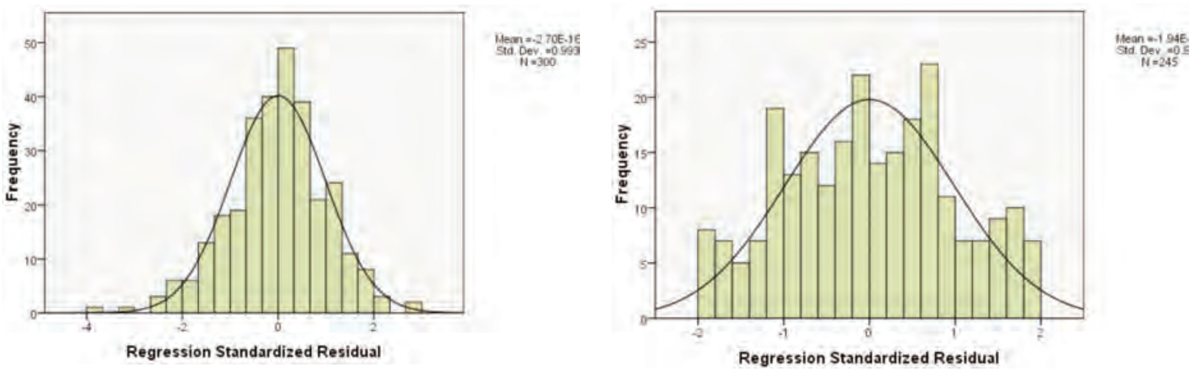


Figure 1 - Linear regression analysis of the Al_2O_3 recovery based on equation (2) for the statistical sample of 300 elements



(a) *(b)*

Fig.2 - The distribution of errors for the initial statistical sample - 300 (a) and for the reduced statistical sample – 245 (b)

With the reduced statistical sample of 245 elements, within the error a limit of 4σ, MLRA was performed using the stepwise method and the results are presented in Table 4.

Obtained results presented in Table 4 show statistical correctness in all cases because VIF<2 and p<0.05. It should be noted that the number of predictors in this case, compared to the previous (sample of 300 elements), increased from 4 to 6 with an increase in R^2 from 0.560 to 0.764. MLRA resulted with the following form of linear dependence of $Y = f(X_i)$:

$$Y = 92.847 - 0.587X_2 + 0.154X_3 - 1.521X_5 - 0.424X_6 - 0.518.X_8 + 6.755X_{10} \quad ...(3)$$

Fig.3. shows the dependence between the measured and calculated values for Y, given by the equation (3), for reduced statistical sample of 245 elements.

Model defined this way, given by the equation (3), has a satisfactory value of

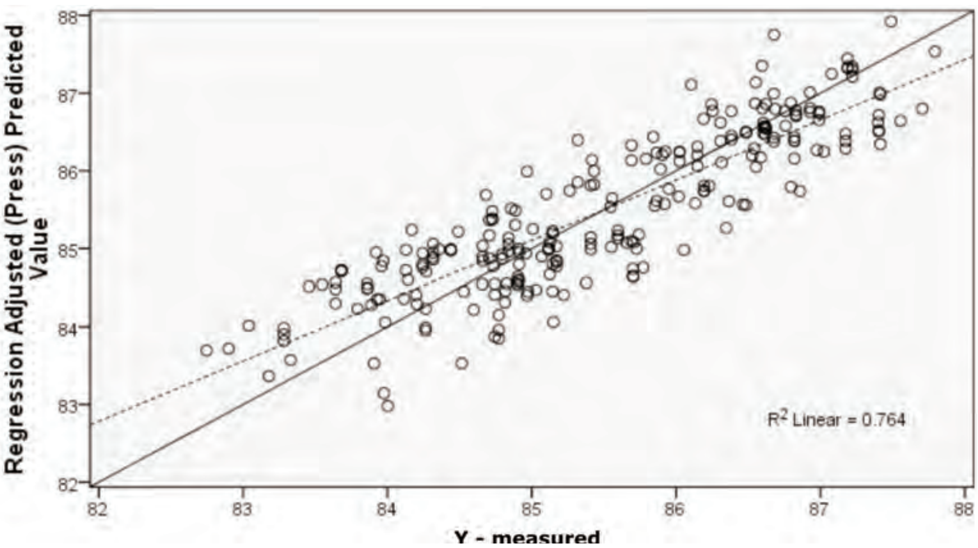


Fig.3. Dependence between the Y - calculated and Y - measured for a reduced statistical sample of 245 elements

coefficient of determination (R^2 = 0.746), and it can be useful for the prediction of the leaching degree of Al_2O_3 with known values of the predictors: caustic module of the initial aluminate solution, the moisture content of bauxite, the content of SiO_2 in the bauxite, the content of Fe_2O_3 in the bauxite, the CaO content in the bauxite and caustic ratio at the end of the leaching process. Parameters with the positive influence on the degree of Al_2O_3 leaching from the bauxite are: caustic module of aluminate solution at the end of the leaching process and the moisture content of bauxite. Following parameters are with the negative influence (in descending order): SiO_2 content in the bauxite, caustic module of aluminate solution at the beginning of the process, CaO content in the bauxite and content of Fe_2O_3 in the bauxite.

Validation of the mathematical model given by the equation (3) was carried out through its application on data obtained in regular production during the following year (2009), using the statistical dataset of 330 measurements. The results of defined model

Table 4. Results of the MLRA for reduced sample of 245 elements, using the Stepwise method.

Model		Unstandard. Coeff.		Stand. Coeff.	t	Sig.	Correlations			Collin. Stat.	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	96.212	0.712		135.103	0.000					
	X5	-1.776	0.117	-0.696	-15.120	0.000	-0.696	-0.696	-0.696	1.000	1.000
2	(Constant)	90.717	0.740		122.557	0.000					
	X5	-1.544	0.096	-0.605	-16.036	0.000	-0.696	-0.718	-0.592	0.957	1.045
	X3	0.359	0.031	0.440	11.664	0.000	0.565	0.600	0.431	0.957	1.045
3	(Constant)	102.005	1.856		54.958	0.000					
	X5	-1.576	0.089	-0.618	-17.699	0.000	-0.696	-0.752	-0.603	0.954	1.048
	X3	0.278	0.031	0.341	8.983	0.000	0.565	0.501	0.306	0.805	1.241
	X6	-0.409	0.063	-0.243	-6.541	0.000	-0.398	-0.388	-0.223	0.841	1.190
4	(Constant)	105.367	1.892		55.682	0.000					
	X5	-1.626	0.085	-0.637	-19.022	0.000	-0.696	-0.775	-0.618	0.941	1.062
	X3	0.245	0.030	0.300	8.085	0.000	0.565	0.463	0.263	0.766	1.305
	X6	-0.487	0.062	-0.289	-7.900	0.000	-0.398	-0.454	-0.257	0.788	1.270
	X8	-0.551	0.110	-0.170	-5.014	0.000	-0.104	-0.308	-0.163	0.916	1.091
5	(Constant)	93.638	2.996		31.259	0.000					
	X5	-1.553	0.083	-0.609	-18.711	0.000	-0.696	-0.771	-0.581	0.911	1.098
	X3	0.193	0.031	0.237	6.271	0.000	0.565	0.376	0.195	0.676	1.478
	X6	-0.447	0.059	-0.265	-7.512	0.000	-0.398	-0.437	-0.233	0.773	1.294
	X8	-0.548	0.105	-0.169	-5.214	0.000	-0.104	-0.320	-0.162	0.916	1.091
	X10	7.740	1.576	0.175	4.910	0.000	0.507	0.303	0.152	0.757	1.320
6	(Constant)	92.847	2.963		31.334	0.000					
	X5	-1.521	0.082	-0.596	-18.442	0.000	-0.696	-0.767	-0.564	0.895	1.118
	X3	0.154	0.033	0.189	4.664	0.000	0.565	0.289	0.143	0.567	1.764
	X6	-0.424	0.059	-0.252	-7.175	0.000	-0.398	-0.422	-0.219	0.759	1.318
	X8	-0.518	0.104	-0.160	-4.979	0.000	-0.104	-0.307	-0.152	0.907	1.102
	X10	6.755	1.590	0.153	4.249	0.000	0.507	0.266	0.130	0.723	1.384
	X2	0.587	0.203	0.114	2.891	0.004	0.533	0.184	0.088	0.600	1.667

validation are shown in Fig.4 with a value of R^2 = 0.759 which is very close to the value determined in the phase of defining the model (training phase) which was 0.764.

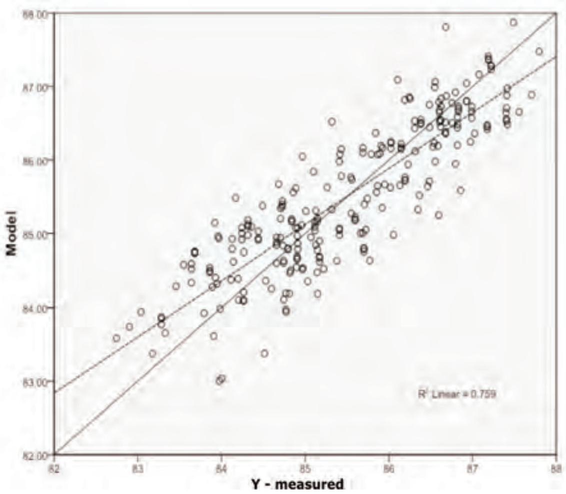


Fig.4. Validation of the model (3) on a sample of 330 measurements in regular industrial production during 2009.

The results of defined linear model validation for the prediction of the degree of Al_2O_3 leaching from the bauxite, in industrial conditions of the Bayer process in the alumina plant BIRAČ in Zvornik, consistently describes 75.9% of the Y variation, depending on the mentioned predictors - the inputs of the process.

4. Conclusion

Stepwise MLRA method was used to define a mathematical model for the dependence of the Al_2O_3 leaching degree from the bauxite in industrial conditions of Bayer alumina process. The investigated production process is facilitated in the alumina factory Birač in Zvornik. Defined linear dependence of the degree of Al_2O_3 leaching on: caustic ratio of the aluminate solution at the beginning and end of the process, the SiO_2, Fe_2O_3, CaO and moisture contents in the bauxite, having a value of R^2 = 0.764 in the stage of defining the model and 0.759 in the model validation phase, indicates the satisfactory degree of fitting of the obtained results.

Further research, aiming to increase the value of R^2, should be focused on applying some of the non-linear regression methods, among which special attention should be paid to the possibility for utilizing the artificial neural networks (ANNs) which, in similar systems, provide much larger values for R^2.[10, 11]

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References

[1] F. Habashi, Handbook of extractive metallurgy, vol.1. Wiley – VCH, Canada

[2] F. Habashi, Journal of Mining and Metallurgy, Section B: Metallurgy, 45 (1) B (2009) 1.

[3] J.A.M. Periera, M.Schwaab, E.Dell'Oro, J.C.Pinto, J.L.F., Monteiro, C.A., Henriques, Hydrometallurgy, 96 (2009) 6.

[4] S. C. Chelgani, E. Jorjani, Hydrometallurgy, 97 (2009) 105.

[5] S. Cao, Y. Zhang, Hydrometallurgy, 98

(2009) 298.

[6] Ž. Živković, N. Mitevska, I. Mihajlović, Dj. Nikolić, Journal of Mining and Metallurgy, Section B: Metallurgy 45 (1) B (2009) 23.

[7] Ž. Živković, I. Mihajlović, Dj. Nikolić, Serbian Journal of Management, 4 (2) (2009) 143.

[8] I. Mihajlović, Dj. Nikolić, N. Štrbac, Ž. Živković, Serbian Journal of Management, 5 (1) (2010) 39.

[9] D.B. Wright, Understanding statistics an: introduction to social sciences, Sage, London 1997, p. 253

[10] P. Djordjević, I. Mihajlović, Ž. Živković, Serbian Journal of Management, 5 (2) (2010) 189.

[11] Ž. Živković, I. Mihajlović, I. Djurić, N. Štrbac, Metallurgical and Materials Transactions 41 B (2010) 1116.