

Letter to Editor

ON THE FIVE BASE QUANTITIES OF NATURE AND SI (THE INTERNATIONAL SYSTEM OF UNITS)

G. Kaptay

*University of Miskolc and BAY-LOGI Research Institute,
Egyetemvaros E/7, 606, Miskolc, Hungary

(Received 20 June 2011; accepted 14 July 2011)

Abstract

It is shown here that five base quantities (and the corresponding five base units) of nature are sufficient to define all derived quantities (and their units) and to describe all natural phenomena. The base quantities (and their base units) are: length (m), mass (kg), time (s), temperature (K) and electric charge (C). The amount of substance (mole) is not taken as a base quantity of nature and the Avogadro constant is not considered as a fundamental constant of nature, as they are both based on an arbitrary definition (due to the arbitrary value of 0.012 kg for the mass of 1 mole of C-12 isotope). Therefore, the amount of substance (mole) is moved from the list of base quantities to the category of the supplementary units (to be re-created after its abrogation in 1995). Based on its definition, the luminous intensity (cd) is not a base quantity (unit), therefore it is moved to the list of derived quantities (units). The ampere and coulomb are exchanged by places in the list of base and derived units, as ampere is a speed of coulombs (but SI defines meter, not its speed as a base unit). The five base quantities are re-defined in this paper by connecting them to five fundamental constants of nature (the most accurately known frequency of the hydrogen atom, the speed of light, the Planck constant, the Boltzmann constant and the elementary charge) with their numerical values fixed in accordance with their CODATA 2006 values (to be improved by further experiments).

Keywords: Base quantity; Base unit; Fundamental constant; SI; International System of Units.

Corresponding author: kaptay@hotmail.com

On the criteria of the ideal base quantities (units) of SI

The International System of Units (SI) today is based on seven base quantities and their corresponding base units [1]: length (m), mass (kg), time (s), temperature (K), electric current (A), amount of substance (mole) and luminous intensity (cd). After 50 years of its successful existence, SI still needs higher precision of its definitions [2]. It might be surprising, but to the opinion of the present author even the number of the seven base quantities (units) of SI should be reconsidered.

The base quantities of any system of units (including SI) should meet the following simple criteria: the amount of base quantities should be as small as possible, but they should allow to define any derived quantity and to describe any natural phenomena. The less base quantities (units) SI has, the less will be to remember for the next generations and the more chances we have that they will capture the essence of SI and will learn how to use it. Therefore it is the responsibility of all scholars to push the SI system into this direction, without losing its power to provide a perfect basis to describe all natural phenomena.

On the five base quantities of nature

It is claimed in this paper that five is the smallest number of base quantities that is sufficient to define all derived quantities and allows the description of all natural phenomena. These five quantities are: length, mass, time, temperature and electric charge. One can see that the first four out of

five in this list coincide with the current list of base quantities in SI. Thus, there is no need to explain why length, mass, time and temperature are base quantities of nature. In the following discussion the reasons why the electric current is replaced by the electric charge and why the amount of substance and luminous intensity are removed from the list of base quantities will be given.

Before going into those details, let us remind that there is nothing new in the above declaration on the five base quantities of nature. The same five base quantities were found by Max Planck, who introduced (as we call them today) the Planck length, the Planck mass, the Planck time, the Planck charge and the Planck temperature [3]. Although Planck used the term “natural units”, in this paper we borrow only his idea about the “natural base quantities”, hidden behind his “natural units”. Planck showed in 1899 that all natural phenomena can be described by his five base quantities (units). To the best of my knowledge, this finding still remains true. The only reason why this paper is written 112 years after the paper of Planck is that SI today has seven base quantities (three of which are different from those found by Planck). The goal of this paper is to convince the engineering and the metrology community to go back and to reorganize SI to be in accordance with the original ideas of Planck. To the opinion of the present author the findings of Planck on the five base quantities (units) is one of the most important findings in the history of natural sciences.

Why to replace electric current by electric charge?

The major reason to do so is to prefer the beauty and the logic of the system of base quantities of SI. One of the undisputable base quantities of SI is length. To my best knowledge length (m) has never been suggested to be replaced by speed (m/s). According to Planck, electric charge (C) is a base quantity of nature. I can see no reason why it should be replaced by its speed ($A = C/s$). Alternatively, if SI prefers A to C, then why SI keeps length (m) in as a base quantity instead of speed (m/s)?

Electric current should be reserved in the list of derived quantities, defined as electric charge per time (similarly to speed, defined as length per time). If we replace the electric current (a base quantity today) by the electric charge (a derived quantity today) by each other, that will change only the theoretical background of SI, without altering its practical side. If we do so, teaching and dissemination of SI will become easier. It is not easy to teach the today's definition of electric current (ampere). On the other hand, electric charge (coulomb) can be easily defined through the elementary charge in a much more straightforward way. Once it is understood, the concept of electric current follows easily using the length – speed analogy.

Why to remove the amount of substance from base quantities?

SI today defines the amount of substance (mole) as one of the base quantities, with the Avogadro constant, treated as a fundamental

constant of nature. However, the amount of substance and the Avogadro constant are defined in an arbitrary way, as the selection of the value of “0.012 kg for 1 mole of C-12” is purely arbitrary. Arbitrary does not mean not convenient: it is based on some clever “numerology”, since “6 protons + 6 neutrons = 12 g/mol” (?).

In his time, Avogadro made a great hypothesis that gases of the same volume, temperature and pressure must consist of the same number of entities (molecules, atoms, etc.). The natural consequence of this hypothesis was the question about the exact number of those entities. Today, however, the number of atoms can actually be counted [4-6], and this is a great achievement.

In real life, the Avogadro constant is used as a grouping constant. Atoms and their masses are so small compared to our human dimensions (to which SI was matched) that there is more convenient to consider them together in groups called mole. The amount of entities in one such group is the Avogadro constant. We should keep them in SI as they are widely used and because they are useful in everyday calculations. However, the fundamental question is not the number of entities in 0.012 kg of C-12. The fundamental question is the mass of 1 atom of C-12 (or Si-28, etc.). If we know the answer to the latter question, we can calculate the mass of any number of C-12 atoms, including that of 1 mole.

Therefore, the amount of substance can be removed from the list of base quantities without any harm. This action will not alter our ability to describe nature. The present author suggests to re-establish (after 1995) the category of supplementary quantities

(units) and to move the amount of substance (mole) from the list of base quantities (units) to the re-established category of supplementary quantities (units). At the same time the value of the Avogadro constant should be fixed forever at its today best value of $6.022\,141\,79\,10^{23}\text{ mol}^{-1}$ [7]. This action changes also the definition of the mole as the amount of substance which contains as many entities as defined by the fixed Avogadro constant. The opposite can be done as well, as the definitions of the mole and that of the Avogadro constant are interconnected in the closed cycle (this is the basic reason why “there is a common misconception among educators and even some metrologists”, as truly pointed by Wheatley [8]). The way out of this closed circle is the above suggestion.

The above does not mean at all that the Avogadro project described in [4] loses its meaning. Mankind still needs to know the as exact as possible values for the mass of Si-28 [5] and the lattice parameter of Si-28 [6] (the latter as function of temperature and pressure). These are the real constants of nature, not the Avogadro constant. Multiplying these values by the fixed Avogadro number, the molar mass and molar volume of different isotopes and elements can be calculated.

Why to remove the luminous intensity from base quantities?

The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of $1/683$ watt per steradian [1]. As follows from this definition, luminous intensity (cd) is based on three derived units (Hz, W, sr), so it is a derived quantity (unit) itself. Therefore, it is suggested here to move this quantity (unit) from the list of base quantities (units) to the list of derived quantities (units). This action will not alter our ability to describe nature and will not cause any confusion at the user side of SI. This action will only make the dissemination of SI easier.

The selected five fundamental constants of nature

Five fundamental constants of nature are selected here with their best available values as suggested by CODATA 2006 [7] (see Table 1). The values of these five base fundamental constants are considered to be fixed. However, their current values should be improved by further experiments. Such

Table 1. The selected five fundamental constants and their best known values [7]

Constant	sign	SI unit	Best known value	u_r
H-frequency*	f_H	s^{-1}	$2.466\,061\,413\,187\,174\,10^{15}$	$1.4\,10^{-14}$
Speed of light	c	m s^{-1}	$2.99\,792\,458\,10^8$	„exact”
Planck constant	h	$\text{kg m}^2 \text{s}^{-1}$	$6.626\,068\,96\,10^{-34}$	$5.0\,10^{-8}$
Boltzmann constant	k	$\text{kg m}^2 \text{s}^{-2} \text{K}^{-1}$	$1.380\,6504\,10^{-23}$	$1.7\,10^{-6}$
Elementary charge	e	C	$1.602\,176\,487\,10^{-19}$	$1.5\,10^{-8}$

* the frequency of $1S1/2 - 2S1/2$ of a hydrogen atom

improvements would strengthen the idea of this paper further.

Our choice in Table 1 coincides in three places with that of Planck (speed of light, Planck constant, Boltzmann constant). The gravitational constant used by Planck is not used here as it has a much higher uncertainty than the constants of Table 1. Planck also used the Coulomb constant to define electric charge. It is in good connection with the existing SI system, as ampere today is defined through the fixed value of the magnetic constant (which follows from the fixed values of electric constant and speed of light, or vice versa). However, to obtain ampere (coulomb) from the electric constant one needs kilogram (the Planck constant), providing higher uncertainty compared to that of the elementary charge (see Table 1). The frequency of Cs-133 used in SI today is replaced by the frequency of the H-atom as it is known by a much higher certainty. Multiplying the Boltzmann constant and the elementary charge of Table 1 by the fixed above Avogadro constant, the universal gas constant and the Faraday constant can be obtained.

The suggested definitions of the five base units

Based on the constants collected in Table 1, the following definitions follow:

- i. second, symbol s, is the unit of time. 1 s is the time during which $2.466\ 061\ 413\ 187\ 174\ 10^{15}$ transitions of $1S_{1/2} - 2S_{1/2}$ type in a hydrogen atom take place ($u_r = 1.4\ 10^{-14}$).
- ii. metre, symbol m, is the unit of length. 1m is the length travelled by light in vacuum

during $(1/2.99\ 792\ 458\ 10^8)$ s, if the speed of light is in SI unit ($u_r = 1.4\ 10^{-14}$).

- iii. kilogram, symbol kg, is the unit of mass. 1 kg is larger by the coefficient of $5.500\ 2390\ 10^{34}$ compared to the numerical value of the expression $h \cdot f_H / c^2$, if all quantities in this expression are in SI units ($u_r = 5.0\ 10^{-8}$).
- iv. Kelvin, symbol K, is the unit of temperature. 1 K is smaller by the coefficient of $1.183\ 521\ 10^5$ compared to the numerical value of the expression, $h \cdot f_H / k$ if all quantities in this expression are in SI units ($u_r = 1.7\ 10^{-6}$).
- v. Coulomb, symbol C, is the unit of electric charge. 1 C is larger by the numerical coefficient of $(1/1.602\ 176\ 487\ 10^{-19})$ compared to the value of the elementary charge, if it is in SI unit ($u_r = 1.5\ 10^{-8}$).

Conclusions

Suggestions to improve the SI system are given in this paper, not changing anything at the user level. The above suggestions improve the theoretical background of the SI system in the following ways:

- i. The suggestions ensure clear distinction between the five base quantities of nature (with their units), the derived quantities (with their units) and the supplementary quantity “amount of substance” (mole), having a historical, but arbitrary chosen definition.
- ii. The suggestions ensure clear distinction between the fundamental constants of nature and the arbitrary chosen value of the Avogadro constant.

- iii. The suggestions ensure long-lasting stability of the five base units, as they are connected to five fundamental constants of nature. Further improvement of measured values of these fundamental constants will gradually improve the accuracy of the base units.
- iv. Teaching and dissemination of SI will become easier if the above suggestions are accepted.
- [8] N. Wheatley Metrologia 48 (3) (2011) 71.

Acknowledgement

This work was carried out as part of the TAMOP-4.2.1.B-10/2/KONV-2010-0001 project with support by the European Union and the European Social Fund.

References

- [1] The International System of Units (SI), 8th edition, 2006. http://www.bipm.org/utis/common/pdf/si_brochure_8_en.pdf
- [2] M.P. Foster Metrologia 47 (6) (2010) R41
- [3] M. Planck in: Proceedings of the Royal Prussian Academy of Sciences in Berlin, 5, 1899 440. (*in German*)
- [4] E. Massa, A. Nicolaus Metrologia 48 (2) (2011) doi:10.1088/0026-1394/48/2/E01.
- [5] A. Picard, P. Barat, M. Borys, M. Firlus., S Mizushima Metrologia 48 (2) (2011) S112.
- [6] E. Massa, G. Mana, L. Ferroglio, E.G. Kessler, D. Schiel, S. Zakel 2011 Metrologia 48 (2) S44.
- [7] P.J. Mohr, B.N. Taylor, D.B. Newell: CODATA recommended values of the fundamental physical constants: 2006. Rev. Modern Phys., 2008, vol.80, pp. 633-730.